Numerical Solution of the Boundary Problem for 2D Laplace Equation for Electrostatic Potential in Given Geometry

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Introduction

The purpose of this project was to further our understanding of computational physics and the implementation of numerical methods in order to study 2D capacitors of various configurations and find space distributions of electric potential, electric field, and charge density. The use of the C programming language was initially chosen because of the group members’ familiarity with it. The experimental grid configuration was adopted from the experiment performed in the introductory physics lab. The grid was segmented and spaced into a thirty by thirty two cells; the extra two were the grounding plates for the configuration. The geometry of the grid consisted of two grounding plates separated by a distance with a rhombus, where the potential was applied, located in the center. The grid spacing was approximately one square centimeter. The computational portion of this project was separated into two sections, the first was creation of the numerical code for the implementation of a finite difference method to solve 2D Laplace equation for a parallel plate capacitor with finite length plates placed inside of a grounded box. The second section was the actual numerical simulation for our experimental configuration.

Method

The Numerical Method

Calculation of Electric Potential

Poisson’s/Laplace’s equation for 2D

\[ \nabla^2 U = 0 \]

Approximated by Taylor expansion

\[ \frac{U_{i,j+1} - U_{i,j}}{\Delta x} + \frac{U_{i+1,j} - U_{i,j}}{\Delta y} = 0 \]

Adding equations for x and y components respectively cancels odd terms and a central difference approximation is obtained

\[ \frac{U_{i+1,j} - U_{i-1,j}}{2\Delta x} + \frac{U_{i,j+1} - U_{i,j-1}}{2\Delta y} = 0 \]

Substituting these into Poisson’s/Laplace’s yields a finite difference form

\[ \frac{U_{i+1,j} - U_{i-1,j}}{2\Delta x} + \frac{U_{i,j+1} - U_{i,j-1}}{2\Delta y} = \frac{\rho}{\varepsilon} \]

If grid spacing is equal

Solving for electric potential \( U \)

\[ \nabla^2 U = \frac{\rho}{\varepsilon} \]

The Experimental Method

Experimental plate was segmented into a 32x30 centimeter area, two sides being grounding strips. The center, were a potential of 10V was applied, of the plate was a rhombus, and was approximated using a step pattern.

Results

Parallel plate Capacitor Potential

Simulation A: parallel plate capacitor were the plates were made from a very thin conductive material with a potential of \( V_1 \) and \( V_2 \).

The program generated expected results for the electric potentials and electric fields. It indicates semi uniform electric field between the plates and maxima at the thin plates with peaks at the corners.

Simulation B: The implementation of charge density applied to the plates which were to be considered a thin dielectric material with uniform charge densities of \( \sigma_+ \) and \( \sigma_- \) on the plates.

Simulation C: Plates considered to be made from a conductive material, as in the first simulation, but with a finite thickness of 2A.

The program generated expected results for the electric potentials with constant values inside of the plates. The electric field showed a marked increase in intensity near the plates and a slight curve around the edges of the plates.

The Simulation of Experimental Results: Electric Field

The graph generated from the experimental data was jagged especially near the center, more points would have smooth this same. The flat boundaries were attributed to the Neumann boundaries of the plate.

Discussion

The program generated expected results for the electric potentials. The electric fields generated were off in unit values, but generated expected shapes. The variation between the simulated and the experimental, was due primarily to difference in boundaries. The simulated results were within a grounded box and the experimental plate consisted of only two grounding sides.

The code was most successful for the Gauss-Seidel method of finite differences. It provided reasonably fast conversion speeds and fair results of the data. The Jacobi method was implemented successfully in the first section for parts a, b, and c, but was slower and less efficient than the Gauss-Seidel method. The Jacobi method provide very similar results to the Gauss-Seidel, there for the Gauss-Seidel method was chosen to implement into section two of the project, the simulation of the experimental plate. The Successive over-relaxation method was experimented with, but it proved to be problematic in its implementation. This was largely a troubleshooting problem, with more time we feel that it would have been implemented effectively and would have provided even faster results than the Gauss-Seidel and the Jacobi methods.

References

Landau Rubi H., Paez Manuel J., Borendas Cristian C., Computational Physics. Problem Solving with Computers