Introduction

The purpose of this research project was to apply knowledge of physics and classical mechanics to analyze the behavior of baseball in flight. More specifically, the projectile motion of a baseball in flight was analyzed to determine which external forces effect the trajectory of the baseball. While the force of the bat and initial angle are important, there are several different factors which also contribute throughout the flight path, such as drag forces. Through the analysis of the drag forces, a Fortran program language code was written, using the RK4 numerical method, capable of predicting the trajectory of a baseball in flight for given initial conditions. Trajectories generated by this numerical model were then compared to those produced through video analysis to confirm the code's reliability.

Theory

Typical analysis of projectile motion problems, presented in any introductory level physics textbook, only accounts for gravitational force. In this case equations of motion can be integrated analytically resulting in well-known parabolic trajectory. However, this approach is only valid for slow moving objects which is no the case for a baseball in flight.

For faster moving objects forces of the air drag and wind are of great importance. Taking wind into consideration, the drag force equation becomes [1]:

\[ F_{\text{drag}} = -b \left( \frac{\rho}{m} \right) v^2 + \text{wind}(v_f + \text{wind}) \]

Here the drag factor, \( b \), is modeled as [1] [2]:

\[ b = \frac{0.0039}{1 + \frac{v^2}{\text{Atmospheric Pressure}}} \]

Where \( v_f \), is the flight speed, \( \rho \), is the density and \( v \). Note that the second the \( \text{wind} \) component is absent in the drag force in the \( y \)-direction because the wind is assumed to only be blowing horizontally. The drag force is also dependent on the air density. This adjustment is incorporated as a ratio between the air density at sea level and the air density at the elevation of the testing site:

\[ F_{\text{drag}} = \rho \frac{P_{\text{sea}}}{P_{\text{elevation}}} \]

However, most barometric pressure readings today include an elevation correction which shows what the pressure would read at sea-level, rather than at the real elevation. To account for this correction, the final formula was determined the atmospheric pressure at the elevation in Abilene, TX [2]:

\[ P_{\text{elevation}} = P_{\text{sea}} \left( \frac{1}{1 + \frac{0.0065}{T - 273.15}} \right) \]

Where \( P_{\text{sea}} \) is the sea-level atmospheric pressure in torr, \( h \) is the altitude in meters, and \( T \) is temperature in degrees Celsius. The corrected pressure was then inserted into the air density equation [3]:

\[ \rho = \frac{1.293 P_{\text{sea}}}{T + 0.00367(T - 298.15) + \text{barometric pressure}} \]

The final force equations are:

\[ \frac{m}{m} - g \frac{\rho}{P_{\text{sea}}} \left( \frac{0.0039}{1 + \frac{v^2}{\text{Atmospheric Pressure}}} \right) v_f + \text{wind} \left( \frac{v_f + \text{wind}}{1 + \frac{v^2}{\text{Atmospheric Pressure}}} \right) \]

Here \( g \) is acceleration due to gravity.

Method

Video Analysis

To acquire the data for the video analysis, a Jugs pitching machine was set up on the first base line of Walt Disney’s Baseball Field. The machine was aimed to shoot at a 45 degree angle in a straight line with first and second base. The slow motion video was filmed at 120 frames per second using an iPhone 5s camera. Prior to each trial the wind speed was measured and recorded using a GLX wind sensor. Each video was then uploaded to the Tracker video analysis software where the ball was marked frame by frame throughout its path. The Tracker software created a table of \( x \) and \( y \) position values which were then used to plot a trajectory graph in MS-Excel.

Numerical Analysis

The fourth-order Runge-Kutta algorithm, or RK4, is a method used to obtain high-precision approximations for the solutions of differential equations. A version of RK4 algorithm was adopted from [4]. The program was written in FORTRAN programming language using the Plato editor, and a compiler provided by SilverFrost. The program was coded to solve the position and velocity differential equations using the RK4 method.

The initial conditions were determined by the trials performed for the video analysis. Using these values for initial position, velocity, and wind speed, the program produced a solution set of \( x \) and \( y \) positions. These values were transferred to MS-Excel and used to plot trajectory graphs.

Results

<table>
<thead>
<tr>
<th>Initial Velocity</th>
<th>Air Density</th>
<th>Altitude</th>
<th>Wind Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.172 m/s</td>
<td>0.994</td>
<td>524 m</td>
<td>4.90 m/s</td>
</tr>
<tr>
<td>Temperature</td>
<td>12.78 C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Discussion

The chart shows the graph of the entire trajectory created by the RK4 program as a part of the numerical analysis as well as the beginning and end of the trajectories produced by the video analysis. Both graphs start at the beginning of the trajectory. At the end of the trajectory, there is a slight discrepancy between the numerical and video analysis plots. The difference between the two graphs could be accounted for by a few possible sources of uncertainty in both the numerical analysis and the video analysis.

To increase the precision of the numerical analysis, the Magnus Force should be added to a source of drag in the kinematic equations. The Magnus Force accounts for the rotation and the drag exerted by the seams on the ball.

A source of uncertainty for the video analysis was the video quality with the use of the Tracker software. To fit the entire path of the ball into one frame, the camera was placed a long distance from the ball itself. This caused the ball to be very small and pixelated, making it a difficult object to track and causing inconsistent data points. Another issue with the video was the visibility of the white ball against the light-blue sky. The lack of color contrast made the tracking of the ball impossible in the upper region of the path. Due to this issue, only the beginning and end of the trajectory from the video was plotted. The skipped steps and missing portion of the trajectory may have caused uncertainty in the data. A final source of uncertainty in the video was the instability of the camera, which caused an inconsistent axis throughout the video. To improve these issues, a higher quality camera should be used along with a tripod to keep the camera stable.

Despite the error in the trajectories provided by the video analysis, the graphs match up well enough to prove that the numerical analysis was successful in predicting the trajectory of a baseball in flight.

RK4 Fortran Program

References