Lecture 5.3

Elasticity

We have looked at many examples of static equilibrium. In all these examples, we have always had 3 unknown variables and 3 equations to solve in order to be able to find these variables. However, there are situations, when the number of unknown variables is more than 3. For instance, in the example about the ladder standing next to the wall, we could take into account the force of friction between the wall and the ladder. In that case we will not be able to solve the problem exactly. The same thing will happen if we consider a table with four legs, trying to find the forces of the legs on the floor. The problem has the exact solution if there are three legs (this is the smallest number of legs to balance the table). In the case of the four leg table it becomes indeterminate. In reality, however, we can measure the force which each of the legs exerts on the floor. The reason why we can not solve this problem is not because the solution does not exist, but because the real tables are not absolutely symmetric with all the legs to be identical, neither those legs nor the tables are perfect rigid bodies. In fact, the legs are deformed slightly under the influence of the table's weight. So, to solve these types of problems we would have to have some knowledge of elasticity.

The real "rigid" bodies are to some extend "elastic", which means they can be deformed by pulling, pushing, twisting or compressing them. However, these effects are very small and in the most part of cases the body comes back to its original state after the external influence has been removed. If, however, the external force was very strong then the body may stay deformed or even be broken. To avoid such a situation is extremely important for engineering practice.

To describe elastic deformations mathematically, we shall introduce two physical quantities. The first is known as stress, which is the force acting per unit of area. The stress produces the strain or unit deformation. There are different types of stresses: 1) tensile stress, due to stretching of a body; 2) shearing stress, due to shearing of the body's layers; 3) hydraulic stress, due to compression of the body by a liquid. As long as the stress and the strain are small enough they are proportional to each other.

\[
\text{Stress} = \text{modulus} \times \text{strain} \tag{5.3.1}
\]
The coefficient of proportionality is called a *modulus of elasticity* and depends on the type of stress. This relation remains linear until the *yield strength* is reached. After that the body becomes permanently deformed. If the stress becomes larger than *ultimate strength* then the body will be broken.

In the case of tension or compression of the object, the stress is defined as the magnitude of tension (compression) force $F$ applied perpendicular to the area $A$, divided by that area $A$. The strain is expressed as fractional change of length, $\Delta L/L$. So the equation 5.3.1 becomes

$$\frac{F}{A} = E \frac{\Delta L}{L},$$

(5.3.2)

where $E$ is known as the Young's elastic modulus. You can find typical values of the Young’s modulus for different substances in Table 12-1 in the book.

In the case of shearing the force vector is in the plane of area $A$, rather than perpendicular to it. The strain is defined as the ratio of the maximum shearing, $\Delta x$, at the top of the body to the height of the body $L$ and equation 5.3.1 becomes

$$\frac{F}{A} = G \frac{\Delta x}{L},$$

(5.3.3)

with $G$ being called the *shear modulus*.

In the case of the hydraulic stress, it is just the liquid's pressure $p$, which deforms the body, and the strain is defined as the fractional change of volume and the equation 5.3.1 becomes

$$p = B \frac{\Delta V}{V},$$

(5.3.4)

where $B$ is known as the *bulk modulus*.

SI unit for stress is Newton per meter squared ($N/m^2$) in all the cases.

You will now have a possibility to measure the Young modulus for specific substance.