Lecture 4.1

Electric Current

So far our study of electricity was limited by electrostatics, the part of physics studying electric charges at rest. However, the majority of electric phenomena, being seen in our everyday life, have to do with moving electric charges. Part of physics which studies motion of electric charges is called electrodynamics.

Many devices we use every day are working based on the principles of electrodynamics. The simplest example will be a flash light. To make it work, one has to have a charged battery, a bulb and a set of wires to connect the bulb with the battery. These are the most important components of any electric circuit. Let us try to understand why we need all those components.

If you perform an experiment with the a bulb, you may find that in order to make this bulb light, you have to obtain the closed circuit which starts at one of the terminals of the battery, it is then connected to one of the terminals of the bulb, and then connected by a wire from the other terminal of the bulb to the other terminal of the battery.

It is necessary to have a closed path in order to allow electric charges to flow from one pole of the battery to another pole of the battery. The battery serves as an energy source for this circuit. A chemical reaction takes place inside of this battery and it divides positive and negative charges. As a result, the potential difference (voltage) is produced between the terminals of the battery. The closed circuit allows charges to flow back from one terminal of the battery to another terminal through electric bulb producing light.

This ordered flow of electric charge is called electric current. According to definition, electric current is the rate at which electric charge is transferred through the cross section area of the wire. Note that we are talking about ordered flow of charge, which is associated with the net transport of charge through the cross section of the wire. Alternatively you can think about chaotic movement of electrons inside of the conductor in the absence of the external electric field. In such a case there is not net transport of charge and there is no current even though the electrons are still moving.

The mathematical definition of current can be written as

$$I = \frac{dq}{dt},$$

(4.1.1)

which is the charge transferred per unit of time. The SI unit of current is Ampere in honor of Andre Marie Ampere (1775-1836) the French physicist, who contributed a lot to the theory of Electromagnetism. The electric current is equal one ampere, if the charge of 1 Coulomb passes the cross section of electric wire in 1 second ($1\ A = 1\ C/s$).
The direction of current is defined as the direction of flow of positive charges. Depending on the type of the conductor, the real current can be associated with motion of either positively or negatively charged particles. In most metals currents occur due to the motion of electrons carrying negative charges. However, the motion of negative charge from the left to the right is equivalent to the motion of the positive charge from the right to the left. So the electric current in metals can be explained as the motion of positive charges but in the opposite direction compared to the motion of the real charged particles (electrons).

One can also introduce the current density, \( \vec{j} \), which has the same direction as velocity of the moving charges and can be interpreted as current flowing per unit area of the cross section of the conductor. In such a case the current itself has a significance of the flux of vector \( \vec{j} \), which is

\[
I = \int \vec{j} \cdot d\vec{A},
\]

where integral is taken through the cross section of the conductor.

The current density is related to the drift velocity, \( \vec{v}_d \), of the positive charges by means of the equation

\[
\vec{j} = ne\vec{v}_d,
\]

where \( n \) is the charge density per unit of volume and \( e \) is an elementary charge.

The flow of current depends on many factors including properties of the battery as well as properties of the conductor. The battery provides potential difference (voltage) to maintain the current. With time chemical processes in the battery will slow down and it will provide less voltage reducing current in the circuit and finally the battery will “die”. The properties of conducting wires or the bulb are also important. Depending on their resistance the current in the circuit can be strong or weak. Very strong current can develop in the circuit with low resistance causing destruction of wires, which occurs in the case of the short circuit.

The flow of electric current is very similar to flow of water in a pipe. The pressure difference at the ends of the pipe is analogous to the potential difference provided by the battery. The higher the pressure difference, the faster (according to Bernoulli’s principle) the liquid in the pipe flows. The thinner pipe provides bigger resistance to water flow than the thicker pipe. This situation is very similar to wires with different resistances and batteries with different potential differences.

Based on this analogy, it is easy to guess that electric current should be higher if the battery provides bigger potential difference and it should be lower if the resistance of the
circuit’s component is high. For simple components of electric circuits, this relation is linear and it is known as Ohm’s law, since it was discovered experimentally by Georg Ohm (1789-1854), which is

The electric current flowing through a given portion of a circuit is equal to the voltage difference across that portion of the circuit divided by the resistance of this portion of the circuit

\[ I = \frac{V}{R} \]  

(4.1.3)

The current is directly proportional to the potential difference and inversely proportional to the resistance.

Your task in the lab today is to check validity of this law.

Ohm’s law also provides a way to define resistance and the SI units for resistance. Resistance is measured in Ohms. One Ohm (1 Ω) is the resistance of the circuit element which has current of 1 A after applying the potential difference of 1 V.

If one looks at the resistive properties of the material at local scale, it also makes sense to introduce resistivity, \( \rho \), of the material. For isotropic materials it can be done by relating the current density with the external electric field as

\[ \vec{E} = \rho \vec{j}, \]  

(4.1.5)

Sometimes the conductivity \( \sigma = \frac{1}{\rho} \) is also introduced, so that

\[ \vec{j} = \sigma \vec{E}, \]  

(4.1.6)

The equation 4.1.6 is sometimes called the local form of the Ohm’s law.

The resistance of each particular circuit element depends on several factors such as conductivity of the element, which is related to the material it is made from, length of the element (the longer wire has greater resistance) and the area of the cross section (the bigger area of the cross section gives less resistance). Combining equations 4.4.4 and 4.1.5 we will have

\[ R = \rho \frac{L}{A}, \]  

(4.1.7)

where \( L \) is the length of the wire, \( A \) is the area of the cross section.

As it was already said, the resistivity depends on the material of the wire. You can find different values of resistivity in table 26.1 on page 689 in the book. Good conductors have low value of resistivity, while for insulators it becomes enormously high. Not only resistivity depends on the material, but it also depends on temperature. Usually it gets larger with increase of temperature. In most part of cases (at least for good metallic
conductor) this dependence is a linear dependence, similar to the one we observed for linear thermal expansion of solids, so

$$\rho = \rho_0 (1 + \alpha \Delta T), \quad (4.1.8)$$

where $\alpha$ is called the temperature coefficient of resistivity and also depends on material. We can use the same idea as in the case of the thermal expansion and build a device, which allows to measure temperature based on resistance changes. This device is called resistance thermometer.

It should also be noted that resistivity of the substance may change with applied electric field, especially if the stronger current causes the higher temperature. So, many real electric devices do not really obey the linear form of the Ohm’s law 4.1.4, since resistance is not constant for them.

One of the main tasks when solving problems is to find current flowing through the circuit (or its parts). In order to find this current, one has to take into account that, in general, a circuit can consist not just of one but of several elements including battery itself and electric wires and other electric devices. All those elements do have resistance, so one needs to know how to calculate combined resistance of this circuit.

Different elements of the circuit can be connected in different ways. Depending on that the current may split at several places (junctions) of the circuit then these currents come back together in other junctions. Therefore, we can distinguish between parallel connection of circuit’s elements and connection of elements in series.

If all the elements are connected in the same loop and the circuit does not have any points, where current can branch into several streams, this connection is called a series connection. In this situation the total resistance of the circuit adds up from individual resistances of the elements, so

$$R_{\text{total}} = R_1 + R_2 + ... \quad (4.1.9)$$

In this case the same current $I$ flows through all the elements of the circuit. At the same time the potential differences (or potential drops) across different elements have to be added up in order to obtain the total voltage drop around the circuit.

The series connection of the elements (let us say electric bulbs) has the important disadvantage. If one of the elements burns out then the entire circuit will be open not allowing any other elements to work properly. That is why another connection, known as parallel connection, is often used. In this situation current splits in junction points into several branches. The sum of all the currents entering junction point is equal to the sum of all the currents leaving this point. On the other hand voltage is the same for all the loops starting and ending at the same points. This means that
\[ \Delta V = \Delta V_1 = \Delta V_2 = ... \]
\[ I = I_1 + I_2 + ... \]
\[ \frac{\Delta V}{R_{\text{total}}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} + ... \]
\[ \frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + ... \]  

(4.1.10)

Your task in the lab today will also be to verify equations 4.1.9 and 4.1.10. To do that you would have to measure voltage and current.

*Voltmeter*, the device which measures voltage between the two points in the circuit is usually connected parallel to the main circuit and it has very large resistance. According to equation 4.1.10, this circuit will have almost the same resistance as if no device was connected at all. This way a voltmeter is not alternating the true value of voltage which it measures.

In a same way *Ammeter*, the device to measure current in a certain part of the circuit has to be connected in series and have almost no resistance, so it will not alternate the value of resistance (equation 4.1.9) and the current which has to be measured.

Now is time to go to the lab and perform all these measurements…