Lecture 2.

Heat Transfer

We have already discussed that if one takes two bodies $A$ and $B$ with original temperatures $T_A$ and $T_B$ and then brings them into contact, the temperature of both bodies will eventually become the same. So, the body which had the higher temperature will be cooled, and another body which had the lower temperature will be heated. This change in temperature occurs due to the energy transfer between the bodies. The internal energy of the bodies will change during this transfer of energy. The transferred energy is called heat and we used letter $Q$ to represent it. The heat is positive when it is absorbed by a body, and it is negative when it is released by the body and transferred to the environment. The important aspect is the existence of the temperature difference. If the temperature difference is already zero, which means the body and the environment are already in equilibrium, then there will be no energy transferred.

Even though we have already discussed transfer of heat between the system and environment, but we never talked about any details of this process. There are three possible transfer mechanisms: conduction, convection and radiation.

Conduction

If you leave one end of the poker in the fire sooner or later the other end of the poker will be heated too. This takes place by means of conduction of heat from one end of the poker to the other end. It occurs because of the interactions between the molecules in the poker. Let us consider a slab of the cross section area $A$ and thickness $L$ with opposite ends maintained at temperatures $T_H$ and $T_C$ by a hot reservoir and a cold reservoir. The heat $Q$ is transferred through this slab from its hot end to its cold end. Experiment shows that the rate of this transfer or conduction rate $P_{cond}$ is $Q/\Delta t$ (the amount of heat transferred per unit of time) is equal

$$P_{cond} = \frac{Q}{\Delta t} = kA \frac{T_H - T_C}{L},$$

(2.3.1)

where constant $k$ is called thermal conductivity. It depends on the material from which the slab is made. Table 18.6 in the book has its values for some materials.

If we are interested in insulation rather than in thermal conductivity, for instance when considering building's walls, we can also introduce another quantity, known as $R$-factor. It is used mostly in engineering practice and defined as
\[ R = \frac{L}{k}. \] (2.3.2)

The lower the \( k \) is the thicker the wall is and the higher its \( R \)-value is going to be, so it is a better insulator. The British unit which is used for \( R \)-values in engineering practice is \( ft^2 \cdot oF/\text{h/Btu} \).

**Example 2.3.1.** Suppose insulating quantities of the wall of a house come mainly from a 4.0 inches layer of brick \((k = 0.84 \, J/s^\circ Cm)\) and R-19 layer of insulation. What is the total rate of heat lost through such a wall, if its total area is 240 \( ft^2 \) and the temperature difference across it is \( 10^\circ F \)?

First of all let us rewrite the equation for the rate of heat transfer in a different form

\[ P_{\text{cond}} = \frac{A\Delta T}{R}, \] where \( \Delta T \) is the temperature difference and \( R = \frac{L}{k} \) is the R-factor. The rate of the heat flow has to be the same for both: the layer of bricks as well as the insulator. If it is not so, then this heat should stay somewhere inside of the wall which is impossible. The temperature, on the other hand, should change smoothly throughout the wall. Let us call \( \Delta T_1 \) to be the temperature change through the brick and \( \Delta T_2 \) the temperature change through the insulator. The total temperature change will be \( \Delta T_1 + \Delta T_2 = \Delta T = 10^\circ F \). So, for the brick one has \( P = \frac{A\Delta T_1}{R_1} \), where

\[
R_1 = \frac{L_1}{k_1} = \frac{4.0in}{0.84\, J/s^\circ C} = \frac{0.33\, ft}{0.84\, J/\text{sm}^\circ C \cdot 4.18 \times 10^3 J/\text{sm}^\circ C \cdot 1^\circ C/3600 \, s} = 0.68 \frac{ft^2\, oF}{Btu}.
\]

is the thermal resistance of the brick wall \((k_1 = 0.84 \, J/\text{ms}^\circ C)\). For the insulator \( P = \frac{A\Delta T_2}{R_2} \)

with \( R_2 = 19 \frac{ft^2\, oF}{Btu} \). This means

\[
R_1 P = A\Delta T_1,
\]

\[
R_2 P = A\Delta T_2,
\]

\[
(R_1 + R_2) P = A(\Delta T_1 + \Delta T_2),
\]

\[
(R_1 + R_2) P = A\Delta T,
\]

\[
P = \frac{A\Delta T}{R_1 + R_2} = \frac{240\, ft^2 \cdot 10^\circ F}{0.68 \frac{ft^2\, oF}{Btu} + 19 \frac{ft^2\, oF}{Btu}} = 1.22 \frac{Btu}{h} = 36W.
\]