Lecture 3.1
Electric Charge and Electric Field

1. Electric Charge

Today we shall start studying the new area of physics, known as Electromagnetism. Until this moment we have only discussed two basic parts of physics: Classical Mechanics and Thermodynamics. The first had to do with motion of objects at macroscopic scale, the second allowed us to learn about exchange of heat and other forms of energy in thermal processes. Of course, physics phenomena are not limited by these two areas only. In our modern world we meet a variety of electric devices almost at every step. Now we have to find out how these devices are working.

As unbelievable as it may sound, Electromagnetism is not that modern branch of science as you may think. Even though people learned how to use electricity for their benefit relatively recent in 19th century, but different types of electric phenomena were known for a long time, starting from the time of ancient Greece or, perhaps, even earlier. The first observations of electric phenomena were related to static electricity rather than to electric current. The part of physics which studies interactions of electric charges at rest is called Electrostatics. It is going to be the subject of our discussion for now. Another subarea of Electromagnetism which studies motion of electric charges is called Electrodynamics. We shall talk about it later in the semester. You may notice that it is very much the same pattern which we used when studied fluids. We first discussed liquids at rest and then we considered more complicated case of flow of fluid.

As it has been mentioned the first experiments with static electric charges were performed in ancient Greece. Even the word “Electrostatics” or “Electricity” comes from the Greek word “electron” which means “amber”, since the first electrostatic experiments were performed with amber. Let us see what these experiments are.

If you run the comb through your hair during a dry day, you can hear cracking sound and sometimes you can even see sparks. After doing that you can notice that the comb will attract small pieces of paper. You can perform similar experiments with amber or plastic if you rub it by a piece of certain fabric. Then a piece of amber or plastic will attract small pieces of paper or pith balls. If we see that there is an interaction between the different objects, this means that there is some kind of force acting between those
objects (let us say plastic rod and pith ball). This force is called electrostatic force and it cannot be described by means of any other forces we studied in classical mechanics. One has to perform several experiments in order to find out what this force depends on.

The most interesting and representative experiment could be performed with two pith balls suspended from the same point by the two strings. If you take a plastic rod rubbed by wool fabric and bring it close to one of the pith balls, the ball will be attracted to the rod until it touches it. If this procedure is performed with both balls, you will see that they are repelled by each other and by the plastic rod mentioned before. At the same time the balls are attracted by a glass rod rubbed with silk. This all brings us to two important conclusions:

1) The ball has received an *electric charge* during its interaction with the rod. It is said, that the ball was charged by *conduction*, since it was in contact with the rod. Since both balls were charged in exact same way, they should have the same charge. The fact that balls repel each other and the rod means that alike charges are repelling.

2) Since the balls are attracted by the glass rod which was charged during rubbing with silk, then there are at least two types of charges in nature and these two different types of charges are attracted.

The charge which appears on a plastic rod when rubbed with wool is called *negative charge*; the charge which appears on glass when rubbed with silk is called *positive charge*. All further experiments never showed existence of any other types of charges in nature. As we saw *charges of the same sign repel; charges of the opposite sign attract*. The question of which of the charges to call positive and which of the charges to call negative is somewhat arbitrary and it is a matter of agreement. Now we are following the convention which was originally introduced in 18th century by Benjamin Franklin. He also introduced the “single-fluid model” to describe the process of charging, which is very close to our modern understanding of this subject. The process of charging occurs when certain amount of the charged substance is transferred from one object to another object. This happens, for instance, during the rubbing of plastic rod by wool. Originally both wool and the rod are neutral. But during the process of rubbing sufficient amount of negatively charged particles known as *electrons* is transferred from the wool to the rod.
As a result, the total charge of the rod becomes negative due to excess of electrons and the total charge of the wool piece becomes positive due to the lack of electrons. During that process the charge is not actually created but just transferred from one substance to another which brings us to another important conclusion, the law of conservation of electric charge.

The total electric charge of the universe is constant. No physical process can result in increase or decrease in the total amount of electric charge in the universe.

Moreover, the transfer of the charge takes place by means of transferring small charged particles. This means that charge is quantized. There is no way to transmit a charge smaller than the charge of one charged particle and all other transmitted charges should be proportional to the charge of that particle. Electron has the smallest possible charge in nature, which is

\[ e = -1.60 \times 10^{-19} \text{C}, \]

(3.1.1)

where \( C \) stands for Coulomb, the SI unit of charge. Charge as well as mass is one of the fundamental attributes of matter, however, even though all elementary particles have nonzero masses but some particles have charges while others do not. In contrast, mass can only be positive (at least as far as we know), while some charges are positive and some are negative.

Talking about properties of different materials relative to ability of transferring charge, experiments show that some substances allow a free flow of charge while other substances do not. Coming back to our example about plastic rod; if you charge one end of this rod you may notice that the other end of the rod is still not charged. This means that plastic does not allow free flow of charge through it, which means that plastic is an insulator. On the other hand if you touch any end of the charged metal rod it will be discharged right away. Most of the metals are good conductors. The difference between conductors and insulators at microscopic level is that the electrons, sometimes called “conduction electrons”, are not bonded by atoms inside of metals but can move freely through entire metal object. At the same time inside of insulators electrons belong to some particular atoms at fixed positions which explain why charge cannot flow.

One can use the property of conductors, allowing free flow of charge, in order to charge a conductor by induction. If you bring the charged object to one side of the neutral
conductor, but do not touch it, the total charge of the conductor is still zero. At the same
time the charge inside of the conductor will be regrouped bringing some of it to one side
of the conductor and some of it to the other. If you touch the opposite (to the charged
object) side of this conductor the excess charge of that side will move to your finger and
the conductor will be charged.

Inside of insulators the picture is rather different. Electrons are attached to certain
atoms fixed in space, but they can still move inside of those atoms. When you bring a
charged object close to the surface of the insulator, electrons will reconfigure their
positions inside of the atoms, forming electric dipoles, where negative charge is slightly
shifted from the positive charge. In this case, we say that insulator becomes polarized.
Some insulators have permanent electric dipoles; those dipoles can change their
orientation in the presence of the external charged object. Some insulators do not have
permanent dipoles and they are formed under the influence of external charged objects.

Let us now study the force between the charged particles. Note that this problem is
very much similar to the problem of gravitational interaction between the two particle-
like objects. Indeed, in both cases there is no direct contact between the interacting
objects, though there is force acting, since both objects are repelled or attracted. In both
cases the force has a long-range nature. It can be detected at very large distance even
though the force is quite small, so one has to apply special techniques in order to measure
it. The technique is the same as we discussed when measuring gravitational force acting
between the two point-like objects. In Cavendish experiment the torsion balance was
used to measure a very small force of gravity. One can use a similar torsion balance to
measure the force acting between the two point-like charges. The experiment shows that
this force depends on the distance between the charges as well as on magnitudes of the
charges. The result is known as the Coulomb’s law by the name of Charles Coulomb
(1736-1806) who first discovered this law.

*The electrostatic force acting between two charged particle-like objects is in the
direction of the line connecting these objects and proportional to the quantity of each of
interacting charges and inversely proportional to the square of the distance between the
charges.*
\[ F = \frac{k|q_1|q_2}{r^2}, \]  

(3.1.2)

where constant \( k = 8.99 \times 10^9 \frac{Nm^2}{C^2} \) is called Coulomb’s constant. This constant is sometimes represented as \( k = \frac{1}{4\pi\varepsilon_0} \), where \( \varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \) is another constant called permittivity of free space. According to Newton’s third law, the same (in absolute value) force acts on the first charge as well as on the second charge, but the two forces are in opposite directions. This law is amazingly similar to the Newton’s law of gravitation. Even though electrostatic force is much stronger force and it can be not just attraction force but the repulsion force as well, the rest of the law is the same. The similarity of the two laws is obvious but so far nobody could find the reason why they are so similar.

Because these two laws are so similar almost everything we have learned about gravitational force is true for electrostatic force. Principle of superposition is working, which means that electrostatic forces created by different particles act as if no other particles exist. To find the net electrostatic force acting on the object, we have to perform vector summation of all electrostatic forces coming from different objects:

\[ \overrightarrow{F_{net}} = \overrightarrow{F_1} + \overrightarrow{F_2} + ... \]  

(3.1.3)

This also means that Newton’s shell theorem is working and the electrostatic force acting between the two spherical uniformly charged objects is the same as if they were particle-like objects with all the charge concentrated at their centers.

**Example 3.1.1.**

Find direction and magnitude of the net electrostatic force exerted on the point charge \( q_3 \) in the figure below, if \( q = 1.8 \times 10^{-6} C \) and \( d=0.22m \). How would your answer change if the distance \( d \) were doubled?
Use Coulomb’s law to find all 3 forces acting on $q_3$.

Directions of these forces are shown on the picture.

$F_{13} = \frac{kq_1q_3}{d^2} = \frac{3kq^2}{2d^2}$

$F_{23} = \frac{kq_2q_3}{d^2} = \frac{6kq^2}{d^2}$

$F_{3} = \frac{kq_3^2}{d^2} = \frac{12kq^2}{d^2}$

Components:

$F_{13x} = -F_{13}\sin 45^\circ = -\frac{3}{2\sqrt{2}} \frac{kq^2}{d^2}$

$F_{13y} = -F_{13}\cos 45^\circ = -\frac{3}{2\sqrt{2}} \frac{kq^2}{d^2}$

$F_{23x} = F_{23}$, $F_{23y} = 0$

$F_{3x} = 0$, $F_{3y} = F_{3}$

$F_{net, x} = \sum F_x = -\frac{3}{2\sqrt{2}} \frac{kq^2}{d^2} + \frac{6kq^2}{d^2} = 3\left(2-\frac{1}{2\sqrt{2}}\right) \frac{kq^2}{d^2}$

$F_{net, y} = \sum F_y = -\frac{3}{2\sqrt{2}} \frac{kq^2}{d^2} + \frac{12kq^2}{d^2} = 3\left(4-\frac{1}{2\sqrt{2}}\right) \frac{kq^2}{d^2}$

$F_{net} = \sqrt{F_{net, x}^2 + F_{net, y}^2} = \sqrt{\left(2-\frac{1}{2\sqrt{2}}\right)^2 + \left(4-\frac{1}{2\sqrt{2}}\right)^2} = 7.2N$

$F_{net} \approx \frac{1}{d}$, so if $d$ changes by 2, $F_{net}$ changes by $2^2 = 4$

Angle between $F_{net}$ and axis x is

$\theta = \tan^{-1}\left(\frac{F_{net, y}}{F_{net, x}}\right) = \tan^{-1}\left(\frac{\sqrt{2}}{2}\right) = 66^\circ$ does not depend on $d$. 

2. Electric Field

Many times during our discussion of gravitational and electrostatic interaction, we mentioned word “field” which is closely related to the long-range nature of gravitational and electrostatic forces. Since both of them do not have direct (by touching) influence on the objects, it is said that they produce a field which is a special form of matter created by the charged (electric field) or massive (gravitational field) objects in the surrounding space.

To see how strong the electric field produced by some charged object is in a given point in space is, one needs to take a testing positive charge \( q \), put it into this point in space and measure the electrostatic force \( \vec{F} \) acting on this testing charge. According to definition, the electric field

\[
\vec{E} = \frac{\vec{F}}{q}
\]

(3.1.4)

The electric field is the electrostatic force acting per unit of positive testing charge. It is a vector and it is measured in \( \text{N/C} \).

So, if we know how strong the electric field in a given point in space is, we can then calculate the force which acts on any charge placed at this point. It is the same as calculating gravitational force applied to a given mass. The gravitational field near the earth’s surface is almost constant and equals \( \vec{g} \) with absolute value of \( 9.8 \text{ m/s}^2 \), so one can find gravitational force as \( \vec{F} = m\vec{g} \). In a same way, if there is a configuration of the charges, producing electrostatic field, one can find the force acting on the testing charge from this field. Principle of superposition works for electric field as well as it works for electrostatic force. For instance (according to the shell theorem) the electric field produced by a uniformly charged sphere outside of this sphere is the same as if all the charge were concentrated at the center of that sphere, so it is the same as electric field of the point-like charge.

According to the Coulomb’s law the electrostatic field produced by one point-like charge \( q \) is

\[
\vec{E} = \frac{kq}{r^2} \hat{r}.
\]

(3.1.5)
It is directed from the charge if it is positive, and towards the charge if it is negative. If we want to use the principle of superposition for electric fields, we can vectorially add various electric fields produced by different point-like charges.

To visualize the net electric field, the concept of electric field lines is used. These lines can be drawn anywhere in space in such a way that electric field in each particular point in space will be in the tangential direction to the electric field lines. There are several simple rules for drawing of the electric field lines. Those lines should

1) point in the direction of the electric field vector \( \vec{E} \) at every point;
2) start at positive charges or at infinity;
3) end at negative charges or at infinity;
4) be most dense where \( \vec{E} \) has a greater magnitude.

Note that electric field lines are not the closed lines. They are always starting at the charged particle and ending at the charged particle. This means, that electrostatic field is a potential field of conservative force, in same way as gravitational field. The fact that electrostatic forces are conservative forces means that the work done by electrostatic force on the moving charge around the closed path is zero and this work does not depend on the form of trajectory but only on the final and original position of the charge.

This knowledge about electric field helps to resolve some problems. For instance if we consider an electric field inside of the charged conductor in electrostatic equilibrium, we can prove that this field is zero. Indeed if we have several charges of the same sign inside of the conductor, they produce electric field, acting on each other and causing them to repel and move until they reach the surface of the conductor and equally distributed over this surface. In this case the electric field lines will start or end at the surface charges, but they can only exist outside of the conductor. Indeed, if there is an electric field inside of the conductor then it should cause motion of charges again, but there is no way for them to move, since they are already on the surface of the conductor and equally distributed and cannot possibly move anywhere further. So that, the field inside of the conductor becomes zero providing electric shielding. This also means that electric charges concentrated on the surface of the conductor may only produce the field, which is directed perpendicular to its surface. Again if it so happens that the field has a
component along the surface then this will course motion of charges along the surface, but they are in equilibrium and cannot move.

As an example on how to use the principle of superposition, let us consider the problem of the electric field created by the two closely positioned point-like charges. This system of the two charges is known as an electric dipole. Let us say, that both these charges have the same absolute value, \( q \), but opposite signs. We shall denote the distance between the charges as \( d \). For now we shall limit our attention by the electric field at point \( P \), which is located on the line connecting the charges at relatively large distance from the interacting charges. We shall place the origin of the coordinate system at the point half way between the charges and choose the \( z \)-axis directed from the negative to the positive charge. In this case the absolute value of the net electric field, produced by the charges at point \( z \) on the \( z \)-axis is

\[
E = E_+ + E_- = \frac{1}{4\pi\varepsilon_0} \frac{q}{r_+^2} - \frac{1}{4\pi\varepsilon_0} \frac{q}{r_-^2} =
\]

\[
\frac{q}{4\pi\varepsilon_0} \left( \frac{1}{\left( z - \frac{d}{2} \right)^2} - \frac{1}{\left( \frac{z + d}{2} \right)^2} \right) =
\]

\[
\frac{q}{4\pi\varepsilon_0 z^2} \left( \frac{1}{\left( 1 - \frac{d}{2z} \right)^2} - \frac{1}{\left( 1 + \frac{d}{2z} \right)^2} \right) =
\]

\[
\frac{q}{4\pi\varepsilon_0 z^2} \left( \frac{1 + \frac{d}{z} + \left( \frac{d}{2z} \right)^2}{\left( 1 - \left( \frac{d}{2z} \right)^2 \right)^2} - \frac{1 + \frac{d}{z} - \left( \frac{d}{2z} \right)^2}{\left( 1 - \left( \frac{d}{2z} \right)^2 \right)^2} \right) = \frac{qd}{2\pi\varepsilon_0 z^3} \left( \frac{2d^2}{z^2} \right) - \frac{1}{\left( \frac{d}{2z} \right)^2}.
\]

If we introduce the electric dipole moment vector \( \vec{p} = q\vec{d} \), directed from the negative to the positive charge and consider the field at a very large distance from the dipole (so that we can ignore small terms of the \( d/z \) order) then the equation for the electric field becomes:
\[
E = \frac{1}{2\pi \epsilon_0} \frac{\rho}{r^2}.
\] (3.1.6)

As we saw in the equation 3.1.5, the electric field produced by a point-like charge is not uniform. To study the effects of potential energy let us construct a uniform field. This is very similar to what we have in the case of gravitational force near the earth’s surface. Let us build a device consisting of two charged plates, known as a \textit{parallel plate capacitor}. If one of the plates is uniformly positively charged and another plate is uniformly negatively charged by the same amount of charge then uniform electric field exists between the plates of this capacitor. The field lines start at positive plate and end at the negative plate going perpendicular to these plates.

So, what is the absolute value of electric field produced inside of the plane capacitor? To answer this question we have to study the electric field produced not by the point-like charges but by the continuous distribution of charge, which we will do next time.