Lecture 3.3

Applications of Gauss' Law

Last time we have introduced the concept of electric flux. According to our definition the electric flux of the electric field $\vec{E}$ through the surface of area $A$ is

$$ \Phi = \int \vec{E} \cdot d\vec{A}, $$

(3.3.1)

where integral is taken over entire surface and, generally speaking, electric field may change from point to point. The $d\vec{A}$ vector is in the direction of external normal to the surface.

We have also proved the Gauss’ Law, which states that the electric flux through any closed surface is equal to the total charge enclosed by this surface divided by permittivity of the free space $\varepsilon_0$.

$$ \Phi = \frac{q}{\varepsilon_0} $$

(3.3.2)

Now, we shall demonstrate how one can use Gauss’ law in order to determine the electric field for various configurations of electric charges. We know, already, that it is sometimes quite a challenging task to calculate such an electric field from the principle of superposition. It is because, it usually involves complicated integration through the volume of the system and we saw that in the examples of electric field of the infinitely long charged wire and infinitely large charged plate. At the same time both these problems possesses cylindrical symmetry which makes them much easier from the standpoint of Gauss’ Law.

If one wants to apply Gauss’ Law as a tool in order to determine the unknown electric field of the system, the first thing to be done is to figure out the symmetry of the problem. Once this is clear, the Gaussian surface can be chosen based on this symmetry. For instance, it can be chosen as a cylinder surrounding infinitely long wire or as a rectangular box with the sides parallel to the infinitely large plane. If the choice was efficient then the unknown electric field will stay constant along such Gaussian surface and will be perpendicular to that surface. This makes integration in equation 3.3.1 trivial and then by combining equations 3.3.1 and 3.3.2, one can find the unknown electric field.

If we are considering infinitely large charged plane or just very large plane, where we neglect any effects that may occur close to the ends of this plane then based on the
symmetry, one can say that the electric field has to be perpendicular to that plane. If the plane is positively charged, then electric field points away from the plane on both sides. Let the plane carry the charge density (charge per unit of area), \( \sigma \). To apply Gauss’ law we have to surround this plane by the closed surface. It could be a parallelepiped with two sides of the area \( A \) parallel to the surface of the plane and all other sides perpendicular to the surface of the plane. The electric field is perpendicular to the plane and so the total electric flux through the surface of this parallelepiped is going to be

\[
\Phi = 2AE = \frac{q}{\varepsilon_0},
\]

so the electric field produced by the charged plane is

\[
E = \frac{q}{2AE_0} = \frac{\sigma}{2\varepsilon_0}
\]

(3.3.3)

Once again, we proved that this field is uniform, but it is much easier way of deriving the equation than by means of integration as we did earlier.

We now consider a device, known as a parallel-plate capacitor, which consists of the two large plates of the same area parallel to each other. Both plates carry the charge of the same absolute value, but one is positively and another one is negatively charged. The total field inside of the capacitor due to both its plates is twice as large as the field of one plate, so the field inside is

\[
E = \frac{\sigma}{\varepsilon_0}
\]

(3.3.4)

The electric field outside of the capacitor is zero because electric fields caused by two different plates have the same magnitude but opposite directions and cancel each other. We shall apply our knowledge of the field inside of the capacitor to consider behavior of electric charges in uniform fields.

We will also consider other examples of applications of Gauss’ Law to the cases of different symmetry in class.