Homework #2

Problems #2.1 and #2.3 from the book.

#1 Consider a system composed of four physically distinguishable particles, each of which may have one of the three possible quantum states with energies $E_1 < E_2 < E_3$.
   a) List the available quantum states for the entire system if it is in contact with heat reservoir. Group the available states according to energy levels.
   b) If the system is now isolated at total energy $E_1 + 2E_2 + E_3$, what are the available energy states?

#2 Consider a system which can contain 0, 1, 2 or 3 particles. In an ensemble representing this system, if $\gamma = -1$:
   a) What is the probability of observing a state with $N=2$?
   b) What is $\bar{N}$?

#3 A grand canonical ensemble is formed, based on a system that can contain 0, 1, 2 or 3 identical particles. Each particle can have an energy $\varepsilon = 1$ or $\varepsilon = 2$. Assume that the particle’s energy is independent of the presence of the other particles.
   a) What are allowed energy levels for the system $E_i(N)$?
   b) If $\gamma = -2$ and $\beta = 1$, what is the probability of finding a system of the ensemble in a state with $N = 3$ and $E_i(N) = 4$?
   c) If $\gamma = -2$ and $\beta = 1$, what is the average number of particles per system of the ensemble?

#4 Develop an expression for the ensemble average volume, $\bar{V} = f(\Omega, V_i)$, for the system of an ensemble based on a prototype system having constant $(N, P/T, E)$, $\Omega$ is the number of available microstates and $V_i$ are the available system volumes consistent with the given $(N, P/T, E)$.

#5 Determine the statistical-mechanical criterion of equilibrium for a $(N, P/T, E)$ system of the previous problem.