Monte Carlo Method

Physics 4362, Lecture #28

Generation of Random Numbers

- No correlations between the numbers
- Uniform
- Non uniform

Random Sequences
Pseudo random number generation

- Linear congruent (power residue) method

\[ r_i = \text{remainder} \left( \frac{ar_{i-1} + c}{M} \right) \]

Example of the code

- Bad
- Good
Assessment of Randomness

The Random Walk Problem

- Results

Radioactive Decay

- Results
Integration by Rejection

- Find $\pi$

Integration by mean value

High-Dimensional Integration

- Results
Integrating Rapidly Varying Functions

- Variance reduction
- Importance Sampling

Ferromagnetism

Spin and Magnetic Moment

\[ \vec{\mu} = g \mu_0 \vec{S} \]
External Field Hamiltonian

\[
\mathcal{H}_0 = -g \mu_0 \sum_{j=1}^{N} \vec{S}_j \cdot \vec{H}_0 = -g \mu_0 H_0 \sum_{j=1}^{N} S_{jz}
\]

Exchange Interaction Hamiltonian

\[
\mathcal{H}_{jk} = -2J \vec{S}_j \cdot \vec{S}_k
\]

Ising Model

\[
\mathcal{H}_{jk} = -2JS_{jz} S_{kz}
\]
Interaction Hamiltonian

\[ H = \frac{1}{2} \sum_{j=1}^{N} \sum_{k=1}^{N} (-2JS_{jz}S_{kz}) = -J \sum_{j=1}^{N} \sum_{k=1}^{N} S_{jz}S_{kz} \]

Weiss Model

Example

Develop expressions for pressure and chemical potential for the Einstein solid, considering \( u_0 \) and \( \theta_0 \) to be functions of \( V/N \). Show that \( \frac{\partial F}{\partial V} \) and \( (F+pV)/N \) give the same expression for chemical potential.