Generation of Random Numbers

**Pseudo random number generation**

- Linear congruent (power residue) method

\[ r_i = \text{remainder} \left( \frac{a r_{i-1} + c}{M} \right) \]
Pseudo Random Number Generation

Assessment of Randomness

The Random Walk Problem

- Results
Radioactive Decay

- Results

Integration by Rejection

- Find $\pi$

Von Neumann Integration Method
Integration by mean value

High-Dimensional Integration

- Results

Integrating Rapidly Varying Functions

- Variance reduction
- Importance Sampling
Ferromagnetism

Spin and Magnetic Moment

\[ \vec{\mu} = g \mu_0 \vec{S} \]

External Field Hamiltonian

\[ \mathcal{H}_0 = -g \mu_0 \sum_{j=1}^{N} \vec{S}_j \cdot \vec{H}_0 = -g \mu_0 H_0 \sum_{j=1}^{N} S_{jz} \]
Exchange Interaction Hamiltonian

\[ H_{jk} = -2J \mathbf{S}_j \cdot \mathbf{S}_k \]

Ising Model

\[ H_{jk} = -2JS_{jz}S_{kz} \]

Interaction Hamiltonian

\[ H = \frac{1}{2} \sum_{j=1}^{N} \sum_{k=1}^{N} (-2JS_{jz}S_{kz}) = -J \sum_{j=1}^{N} \sum_{k=1}^{N} S_{jz}S_{kz} \]
The Metropolis Algorithm

- Start with an arbitrary configuration of spins
- Generate Trial configuration
- Calculate the energy of the trial configuration
- Compare the energies
- Accept or do not accept the new configuration

Weiss Model