Monte Carlo Method

Physics 4362, Lecture #30

Generation of Random Numbers

The Random Walk Problem

- Results
Radioactive Decay

Results

Integration by Rejection

Find $\pi$

Von Neumann Integration Method
Integration by mean value

High-Dimensional Integration

- Results

Ferromagnetism
Spin and Magnetic Moment

\[ \vec{\mu} = g \mu_0 \vec{S} \]

External Field Hamiltonian

\[ \mathcal{H}_0 = -g \mu_0 \sum_{j=1}^{N} \vec{S}_j \cdot \vec{H}_0 = -g \mu_0 H_0 \sum_{j=1}^{N} S_{jz} \]

Exchange Interaction Hamiltonian

\[ \mathcal{H}_{jk} = -2J \vec{S}_j \cdot \vec{S}_k \]
Ising Model

\[ \mathcal{H}_{jk} = -2JS_{jz}S_{kz} \]

Interaction Hamiltonian

\[ \mathcal{H} = \frac{1}{2} \sum_{j=1}^{N} \sum_{k=1}^{N} (-2JS_{jz}S_{kz}) = -J \sum_{j=1}^{N} \sum_{k=1}^{N} S_{jz}S_{kz} \]

The Metropolis Algorithm

- Start with an arbitrary configuration of spins
- Generate Trial configuration
- Calculate the energy of the trial configuration
- Compare the energies
- Accept or do not accept the new configuration