Transport Theory

Kinetic theory of Gases

- Large number of molecules in macroscopic volume
- Big distances between molecules
- Continuous motion of molecules
- Elastic collisions
Molecular Distribution Function

\[ f(\vec{r}, \vec{v}, t) \]

Properties of Gas

\[ \langle Z(\vec{r}, t) \rangle = \frac{1}{n(\vec{r}, t)} \int d^3\vec{v} f(\vec{r}, \vec{v}, t) Z(\vec{r}, \vec{v}, t) \]

Flux of physical quantity

\[ F_n(\vec{r}, t) \]
Flux of physical quantity

\[ F_n(\vec{r}, t) = \int d^3\vec{v} f(\vec{r}, \vec{v}, t) \hat{n} \cdot \vec{U}(\vec{r}, \vec{v}, t) = n(\vec{r}, t) \langle \hat{n} \cdot \vec{U} \chi \rangle \]

Boltzmann Equation in the Absence of collisions

\[ \frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} + \frac{\vec{F}}{m} \cdot \frac{\partial f}{\partial \vec{v}} = 0; \]
\[ Df = 0 \]

Effects of Collisions
Path Integral formulation of Transport problem

\[ f(\vec{r},\vec{v},t) = \int_0^\infty f^{(0)}(\vec{r}_0,\vec{v}_0,t-t') e^{-i\tau} \frac{dt'}{\tau} \]

Example: Electrical Conductivity

\[ \Delta f = f(\vec{r},\vec{v},t) - f^{(0)}(\vec{r},\vec{v},t) = \int_0^\infty \frac{df^{(0)}(\vec{r}_0,\vec{v}_0,t-t')}{dt'} e^{-i\tau} dt' \]
\[ f(\vec{r}, \vec{v}, t) = g(\varepsilon) - eE\tau_v \frac{dg}{d\varepsilon} \]

Conductivity for ions and low concentrations of electrons

\[ \sigma = \frac{j}{E} = \frac{ne^2}{\tau} \]

Conductivity for electrons

\[ \sigma = \frac{j}{E} = \frac{ne^2}{\tau_F} \]
Example: Viscosity

\[ f(\vec{r}, \vec{v}, t) = f^{(0)}(\vec{r}, \vec{v}, t) + \frac{du_x}{dz} \frac{dg}{dU_x} v_z \tau \]

Coefficient of Viscosity

\[ \eta = nkT \bar{\tau} = \frac{1}{3} nm \bar{\tau} U^2 \approx \frac{1}{3} nml \bar{v} \]
Boltzmann differential equation
and collisions

\[ Df = \frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} + \frac{\vec{F}}{m} \cdot \frac{\partial f}{\partial \vec{v}} = - \frac{f - f^{(0)}}{\tau_0} \]

Equivalence of the two
formulations
Applications of Boltzmann equation

\[ f = f^{(0)} + f^{(1)} \]