Statistical Thermodynamics

Ensemble
Random Walk Problem

The probability of taking $N$ steps with $n_1$ steps to the right and $n_2 = N - n_1$ to the left is given by:

$$W_N(n_1) = \frac{N!}{n_1! n_2!} p^{n_1} q^{n_2}$$

where $p$ is the probability of moving to the right and $q = 1 - p$ is the probability of moving to the left.

The probability $P_N(m)$ of reaching the $m$th step in $N$ steps is given by:

$$P_N(m) = \binom{N}{m} \left( \frac{N+m}{2} \right) \left( \frac{N-m}{2} \right) \frac{N!}{m! (N-m)!} p^{N+m} (1-p)^{N-m}$$
The mean value of discrete variable

\[ \bar{u} = \frac{\sum_{i=1}^{M} P(u_i)u_i}{\sum_{i=1}^{M} P(u_i)} \]

Moments of the distribution

\[ \bar{f}(u) = \sum_{i=1}^{M} P(u_i)f(u_i) \]
Random walk problem calculations

Probability distribution for large N

Gaussian Distribution

\[ W(n_i) = \left[ \frac{2\pi}{\Delta n_i} \right]^{3/2} \exp\left[ -\frac{(n_i - \bar{n}_i)^2}{2(\Delta n_i)^2} \right]. \]

\[ W(n_i) = \left[ \frac{2\pi}{Npq} \right]^{3/2} \exp\left[ -\frac{(n_i - Np)^2}{2Npq} \right]. \]
Gaussian Distribution

\[ P(m) = \left[ 2\pi Npq \right]^{1/2} \exp \left[ -\frac{(m-N(p-q))^2}{8Npq} \right] \]

Gaussian Distribution

\[ p(x) dx = \frac{1}{\sqrt{2\pi \sigma}} \exp \left[ -\frac{(x-\mu)^2}{2\sigma^2} \right] dx, \]

\[ \mu \equiv (p-q)Nl, \]
\[ \sigma \equiv 2\sqrt{Npq}l \]

The case of several variables
Statistically Independent Variables

\[ P(u_i, v_j) = P_u(u_i) P_v(v_j) \]

Mean Values

\[ F(u, v) = \sum_{i=1}^{M} \sum_{j=1}^{N} P(u_i, v_j) F(u_i, v_j) \]

Continuous Probability Distribution
Continuous Probability Distribution in the case of several variables

General Random Walk Problem