Random Walk Problem

Probability of taking $N$ steps with $n_1$ steps to the right and $n_2 = N - n_1$ to the left

$$W_N(n_1) = \frac{N!}{n_1!n_2!} p^{n_1} q^{n_2}$$
\[ P_n(m) = \binom{N+m}{N} \binom{N-m}{2} p^n (1-p)^{N-m} \]

The mean value of discrete variable

\[ \bar{u} = \frac{\sum_{i=1}^{M} P(u_i) u_i}{\sum_{i=1}^{M} P(u_i)} \]

\[ \bar{f}(u) = \sum_{i=1}^{M} P(u_i) f(u_i) \]
Moments of the distribution

Probability distribution for large N

Gaussian Distribution

\[ W(n_i) = \left[ 2\pi (\Delta n_i)^2 \right]^{-1/2} \exp \left[ -\frac{(n_i - \bar{n}_i)^2}{2(\Delta n_i)^2} \right], \]

\[ W(n_i) = \left[ 2\pi Npq \right]^{-1/2} \exp \left[ -\frac{(n_i - Np)^2}{2Npq} \right]. \]
Gaussian Distribution

\[ P(m) = [2\pi Npq]^{-1/2} \exp\left[ -\frac{(m - N(p - q))^2}{8Npq} \right] \]

Gaussian Distribution

\[ \mathcal{P}(x) dx = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[ -\frac{(x - \mu)^2}{2\sigma^2} \right] dx, \]

\[ \mu \equiv (p - q)Nl, \]

\[ \sigma \equiv 2\sqrt{Npq}l \]

The case of several variables
Statistically Independent Variables

\[ P(u_i, v_j) = P_u(u_i) P_v(v_j) \]

Mean Values

\[ F(u, v) = \sum_{i=1}^{M} \sum_{j=1}^{N} P(u_i, v_j) F(u_i, v_j) \]

Continuous Probability Distribution
Continuous Probability Distribution in the case of several variables

General Random Walk Problem